MATH 140A Review: Inequalities and Absolute Values

Facts to Know:

Properties of inequalities:

1. **(addition)** $a \le b$ iff $a + \varepsilon \le b + \varepsilon$
2. **(multiplication by $\epsilon > 0$)** $a \le b$ iff $a \cdot \varepsilon \le b \cdot \varepsilon$
3. **(multiplication by $\epsilon < 0$)** $a \le b$ iff $a \cdot \varepsilon \ge b \cdot \varepsilon$

Properties of absolute value:

1. The absolute value of x is defined by $|x| = \begin{cases} x & , \text{if } x \ge 0 \\ -x & , \text{if } x < 0 \end{cases}$
2. $|x| < \epsilon$ iff $-\varepsilon < x < \varepsilon$
3. $|a \cdot b| =$ $|a| \cdot |b|$
4. Does $|a + b| = |a| + |b|$ generally hold? $0 = |1 - 1| \ne |1| + |1| = 2$
5. $|a + b| \le$ $|a| + |b|$

Example: Let $n = 1, 2, \ldots$. Determine for what values of n the following holds

$$\left| \frac{7n+5}{3n-4} - \frac{7}{3} \right| < \frac{1}{2020}. \qquad (*)$$

Solution. $(*)$ holds iff

$$-\frac{1}{2020} < \frac{7n+5}{3n-4} - \frac{7}{3} < \frac{1}{2020}$$

iff

$$\frac{-1}{2020} + \frac{7}{3} < \frac{7n+5}{3n-4} < \frac{1}{2020} + \frac{7}{3}. \quad (**)$$

If $n = 1$, then $(*)$ is false. If $n \ge 2$, then $3n-4>0$. Thus, (**) holds iff

$$\frac{-(3n-4)}{2020} + 7n - \frac{7\cdot 4}{3} < 7n+5 < \frac{3n-4}{2020} + 7n - \frac{7\cdot 4}{3}$$

iff

$$\frac{-(3n-4)}{2020} < 5 + \frac{7\cdot 4}{3} < \frac{3n-4}{2020}.$$

Since $\frac{-(3n-4)}{2020} < 0 < 5 + \frac{7\cdot 4}{3}$, then we only need to focus on:

$$5 + \frac{7\cdot 4}{3} < \frac{3n-4}{2020}$$

iff

$$9652.4 \approx \left[\left(5+\frac{7\cdot 4}{3}\right)2020+4\right]/3 < n.$$

Thus, (*) holds iff $n \geq 9653$.

$\square$